12.5
12.6
13.1

$$\lim_{x \to a} f(x) = L$$

For $\forall \varepsilon$, $\exists\ \delta$ where

if $0 < |x-a| < \delta$, then $0 \le |f(x) - L| < \varepsilon$.

1) $\lim_{x \to 4} 2x - 5 = 3$

S.W.

$f(x) = 2x-5$

$\boxed{|x-4| < \delta}$

$\boxed{2|x-4| < \varepsilon}$

$|2x-8| < \varepsilon$

if $\delta = \frac{\varepsilon}{2}$

$|2x-5-3| < \varepsilon$

$|x-4| < \frac{\varepsilon}{2}$

To show that $\lim_{x \to 4} 2x-5 = 3$

Given: $\varepsilon > 0$

Define that $\delta = \frac{\varepsilon}{2}$

Assume $|x-4| < \delta$, we need to show that

$$|(2x-5)-3| < \varepsilon.$$

$|x-4| < \delta$

$2|x-4| < 2\delta$

$|2x-8| < 2\delta$

$|(2x-5)-3| < 2\delta = 2\left(\frac{\varepsilon}{2}\right) > \varepsilon$

$\therefore \lim_{x \to 4} 2x-5 = 3$

Prove that

$$\lim_{x \to 2} 3x-1 = 5$$

Scratch

① $|x-2| < \delta$

② $|3x-1-5| < \varepsilon$

$|3x-6| < \varepsilon$

$3|x-2| < \varepsilon$

$|x-2| < \frac{\varepsilon}{3}$

if $\delta = \frac{\varepsilon}{3}$

Given: $\varepsilon > 0$

Defined that $\delta = \frac{\varepsilon}{3}$

$|x-2| < \delta$

$3|x-2| < 3\delta$

$|3x-6| < 3\delta$

$|3x-1-5| < 3\delta = 3\left(\frac{\varepsilon}{3}\right) = \varepsilon$

therefore $\lim_{x \to 2} 3x-1 = 5$

$$\lim_{x \to 3} x^2 - 1 = 8$$

$|x-3| < \delta$

$|x^2 - 1 - 8| < \varepsilon$

$|(x-3)(x+3)| < \varepsilon$

$|(x+3)|\,|x-3| < \varepsilon$

$|x-3| < \dfrac{\varepsilon}{x+3}$